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RB-0726

Second Year B. Sc. (I.C.) Examination

April / May – 2010

Mathematics : Paper - III

(New Course)

Time : **3** Hours] [Total Marks : **105**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ S.Y. B.Sc. (I.C.)

Name of the Subject :

☛ MATHEMATICS - 3 (NEW)

☛ Subject Code No. : 0 7 2 6 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) All the questions are compulsory.

(3) Digits to the right indicate marks of question.

(4) Follow usual notations.

1 Answer the following : **15**

(1) Define maximum value of a function.

(2) Obtain the degree of non-homogeneous function

$$f(x, y) = \frac{x^{1/4} + y^{1/4}}{x^{1/8} + y^{1/8}}$$

(3) Find particular integral of $\left(D^2 - 1\right) y = \cos ax$.

(4) If $x = r\cos\theta$ and $y = r\sin\theta$ then find $\frac{\partial(x, y)}{\partial(r, \theta)}$.

(5) State Euler's theorem for non-homogeneous function of two variables.

2 (a) If $u = \tan^{-1}\dfrac{x^3+y^3}{x+y}; x+y \neq 0$ then prove that **6**

(i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$ and

(ii) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u$

(b) Discuss the continuity of $f(x, y)$ at point $(0, 0)$ **6**

where $f(x, y) = \dfrac{xy}{\sqrt{x^2+y^2}}$ where $(x, y) \neq (0, 0)$

$= 0$ where $(x, y) = (0, 0)$

(c) In usual notation prove that, **6**

$dz = \dfrac{\partial z}{\partial x}dx + \dfrac{\partial z}{\partial y}dy$

OR

2 (a) Show that **6**

$e^{ax}\sin by = by + abxy + \dfrac{1}{6}\left(3a^2bx^2y - b^3y^3\right) + ...$

(b) If $u = \sin^{-1}\dfrac{x^2+y^2}{x+y}$ where $x+y \neq 0$ then prove that **6**

$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$

(c) Discuss the limit of $f : R^2 \to R$ at the point $(1, 2)$ **6**
where

$f(x, y) = 3xy \quad ; \quad (x, y) \neq (1, 2)$

$= n \quad ; \quad (x, y) = (1, 2)$

3 (a) Find extreme values of the function **9**

$f(x, y) = x^3 + y^3 - 3axy$

(b) After changing the order of integration evaluate **9**

$$\int_0^1 \int_y^{2-y} \left(x^2 + y^2\right) dxdy$$

OR

3 (a) Find the point $p(x, y)$ from which the sum of the distance from the axes and the line $x + y = 8$ is minimum. **9**

(b) Find the value of **9**

$$\int_1^2 \int_2^3 \left(y^2 + 2xy\right) dxdy$$

4 (a) Derive the method of $\frac{1}{f(D)} e^{ax}$ when $f(a) = 0$. **8**

(b) Solve : **10**

(i) $\left(x^2D^2 + 2xD - 12\right) y = (x+1)^2$

(ii) $\frac{d^3y}{dx^3} + y = 3 + e^{-x} + 5e^{2x}$

OR

4 (a) In usual notation prove that, **8**

$$\frac{1}{\phi\left(D^2\right)} \sin ax = \frac{1}{\phi\left(-a^2\right)} \sin ax; \phi\left(-a^2\right) \neq 0$$

(b) Solve : **10**

(i) $\frac{d^3y}{dx^3} - y = \left(e^x + 1\right)^2$

(ii) $\left(D^2 - 2D + 1\right) y = x^2 + 1$

5 (a) In usual notation prove that 8

$$\frac{1}{f(D)}e^{ax}\cdot V = e^{ax}\cdot\frac{1}{f(D+a)}V$$

(b) Solve : 10

(i) $\left(D^3-3D^2+4\right)y = e^{3x}$

(ii) $\left(D^2+4\right)y = \sec 2x$

OR

5 (a) Discuss how to solve partial differential equations of type $f_1(x,p) = f_2(y,q)$ 8

(b) Solve : 10

(i) $q = xyp^2$

(ii) $\left(x^2-y^2\right)p+\left(y^2-z^2\right)q = z^2-x^2$

6 (a) Discuss how to solve partial differential equations of the type $F(z,p,q)=0$ 8

(b) Solve : 10

(i) $xzp+yzq = xy$

(ii) $z^2\left(p^2+q^2+1\right) = c^2$

OR

6 (a) Explain the method of solving the equation of type $F(p,q)=0$ 8

(b) Solve : 10

(i) $z = px+qy-2\sqrt{pq}$

(ii) $\left(x^2-yz\right)p+\left(y^2-zx\right)q = z^2-xy$